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# A Method for the Probabilistic Design Assessment of Composite Structures

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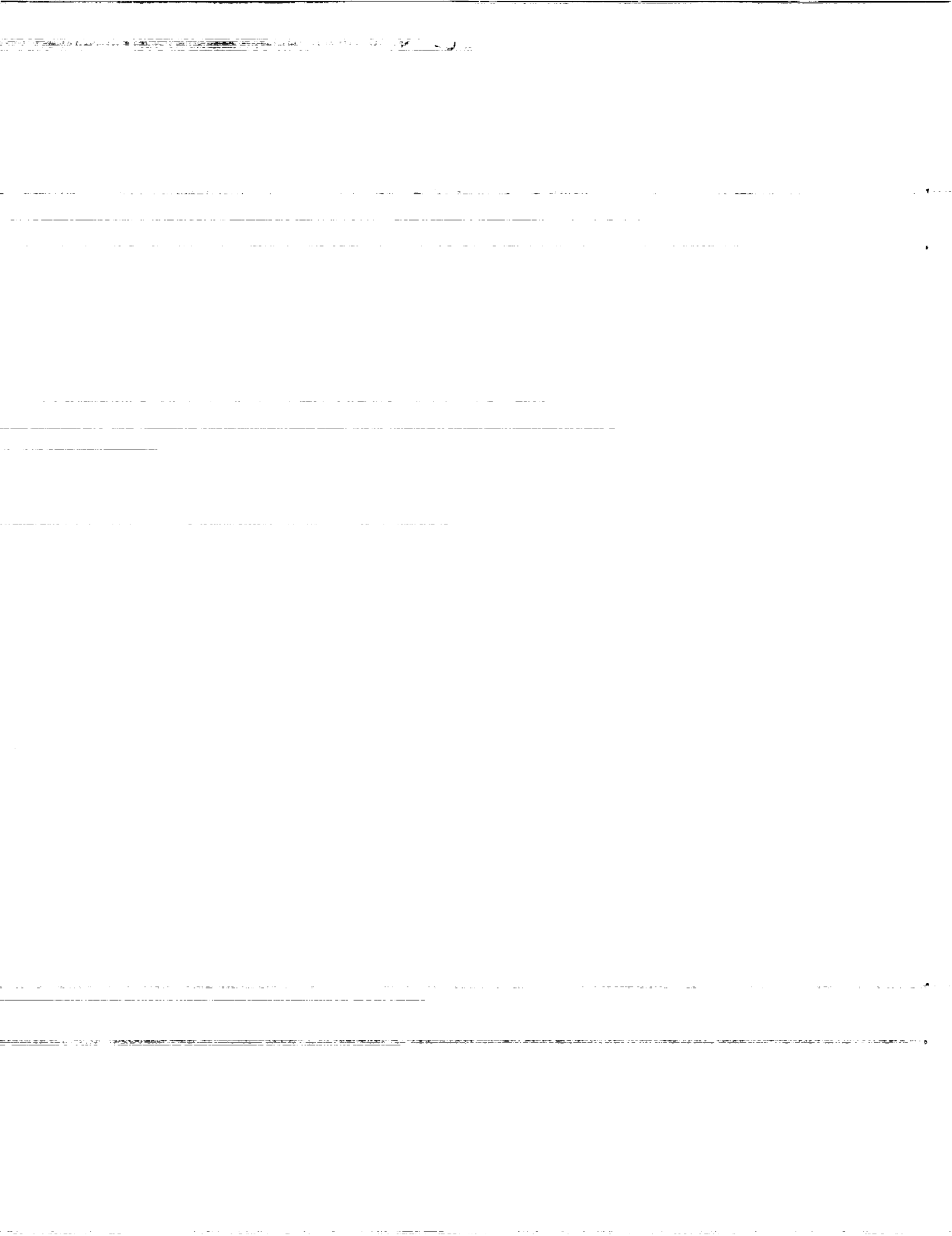
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# A METHOD FOR THE PROBABILISTIC DESIGN ASSESSMENT OF COMPOSITE STRUCTURES

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## SUMMARY

A formal procedure for the probabilistic design assessment of a composite structure is described. The uncertainties in all aspects of a composite structure (constituent material properties, fabrication variables, structural geometry, and service environments, etc.), which result in the uncertain behavior in the composite structural responses, are included in the assessment. The probabilistic assessment consists of (1) design criteria, (2) modeling of composite structures and uncertainties, (3) simulation methods, and (4) the decision-making process. A sample case is presented to illustrate the formal procedure and to demonstrate that composite structural designs can be probabilistically assessed with accuracy and efficiency.

## INTRODUCTION

Composite materials are widely used in modern structures for high performance and reliability. However, because these structures usually operate in hostile and random service environments, it is difficult to predict the structural performance. In addition, experiments show that the composite structural behavior exhibits wide scatter as a result of the inherent uncertainties in design variables. The design variables, known as primitive variables, include the fiber and matrix material properties at the constituent level; fiber and void volume ratios, ply misalignment and ply thickness for the fabrication process; and random structure size, loadings, and temperature.

The scatter in the structural behavior cannot be computationally simulated by the traditional deterministic methods that use a safety factor to account for uncertain structural behavior; thus, the structural reliability cannot be discerned. A probabilistic design methodology is needed to accurately determine the structural reliability of a composite structure. In the past, Monte Carlo methods have been widely used for probabilistic composite structural analysis (ref. 1). However, these methods are computationally intensive and can be used for verification purposes only. Likewise, when perturbation techniques (ref. 2) are used to account for uncertainties, only the first few statistical moments of the structural responses — and not the cumulative distribution function (CDF) — are obtained.

NASA Lewis Research Center has developed a formal methodology to efficiently and accurately quantify the scatter in the composite structural response and to assess the composite structural design, while accounting for the uncertainties at all composite levels (constituent, ply, laminate, and structure) (fig.1). This methodology, which integrates micro- and macrocomposite mechanics and laminate theories, finite element methods, and probability algorithms, was implemented through the computer code IPACS (Integrated Probabilistic Assessment of Composite Structures) (fig. 2) (ref. 3). IPACS is used to assess composite structures probabilistically for all types of structural performances such as instability, clearance, damage initiation, delamination, microbuckling,

fiber crushing, and resonance damage. Since IPACS uses a special probability algorithm FPI (fast probability integrator) (ref. 4) instead of the conventional Monte Carlo simulation, tremendous computational time can be saved (ref. 5). Therefore, a probabilistic composite structural analysis, which cannot be done traditionally, becomes desirable especially for large structures with many uncertain variables. A typical case is analyzed herein to demonstrate the code IPACS for the probabilistic assessment of composite structures and to illustrate the formal design assessment methodology.

## FUNDAMENTAL APPROACH

The fundamental approach for the probabilistic assessment is as follows:

- (1) Identify all possible uncertain variables at all composite scale levels.
- (2) Determine the probabilistic distribution function (PDF) for each variable.
- (3) Process all random variables through an analyzer that consists of micro- and macrocomposite mechanics and laminate theories, structural mechanics, and probability theories.
- (4) Extract useful information from the output of the analyzer and check against the probabilistic design criteria.

The uncertainties in a composite structural design can originate at different composite scale levels. At the constituent level, the material properties for the fiber and matrix are the major sources of uncertainties. The primitive variables are defined in the appendix and their typical values are listed in table I. At all stages of the fabrication process, the fabrication variables such as fiber volume ratio, void volume ratio, ply misalignment, and ply thickness show considerable scatter. At the structure level, variation of the geometry during the assembly stage, uncertain boundary conditions, and random thermal-mechanical loads contribute significantly to the scatter in the composite structural response.

Once the uncertainties in the primitive variables are identified, micro- and macrocomposite mechanics and laminate theories are used to propagate them in the constituent material properties and in the fabrication variables from the lower composite levels (ply) to the higher composite levels (laminate). The scatter in the structural response is then computationally simulated through structural mechanics (or finite element methods) and through probability theories (Monte Carlo simulation or FPI) to account for uncertainties in the laminate material properties, geometry, boundary conditions, and thermal-mechanical loads.

Probabilistic design assessment includes probabilistic design criteria that are based on considerations such as safety, performance, economics, and other requirements that could introduce uncertainties in a specific design. These factors contribute to the uncertainties of the structural response and provide inclusive information for judicious decisions on design acceptance or rejection.

## PROBABILISTIC COMPOSITE STRUCTURAL DESIGN ASSESSMENT

This report describes a probabilistic design assessment methodology for composite structures with an acceptable or preassigned reliability. The details of the assessment are described in the following three steps.

## Step 1: Probabilistic Design Criteria Setup

A typical design criterion can be stated as follows: The probability of a failure event should be less than an acceptable value, say  $10^{-3}$ . A failure event occurs when a structural response is less than the allowable response. This probability is defined as the failure probability. The allowable response divides the possible response domain into safe and failure regions as shown in figure 3. The predicted failure probability is the area under the probability density function in the failure region. The critical response (fig. 3) is determined by IPACS such that the probability of a response exceeding this critical value is in the safe region. When the critical response falls within the safe region, the design is acceptable. When the critical response falls within the failure region, the design is unacceptable and requires a redesign. Sample probabilistic design criteria for the various failure modes are described as follows:

- (1) Instability—The probability that the buckling load is smaller than the design load should be less than  $10^{-3}$ .
- (2) Clearance—The probability that the nodal displacement is greater than the allowable tolerance should be less than  $10^{-3}$ .
- (3) Resonance avoidance—The probability that the natural frequency is greater than its upper bound should be less than  $10^{-3}$ .
- (4) Delamination—The probability of delamination occurrence should be less than  $10^{-3}$ .

## Step 2: Probabilistic Simulation Using IPACS

IPACS integrates several NASA in-house computer codes developed in recent years such as COBSTRAN (Composite Blade Structural Analyzer) (ref. 6), PICAN (Probabilistic Integrated Composite Analyzer) (ref. 7) and MHOST (Marc Hot Section Technology) (ref. 8). COBSTRAN is a dedicated finite element model generator for structural analysis of composite structures. PICAN uses ICAN (Integrated Composite Analyzer) computer code (ref. 9) for composite mechanics. This code has evolved over the last 20 years and has been verified with experimental data for all aspects of composites. PICAN enables the computation of the perturbed and probabilistic composite material properties at the ply and laminate levels. MHOST performs structural analyses using verified finite element methods. These analyses determine the perturbed and probabilistic structural response at global, laminate, and ply levels. PICAN and MHOST share the FPI module (ref. 4) for the application of the fast probability integration algorithm to obtain cumulative distribution functions (CDF's) of the material properties and the structural responses.

FPI is an approximate technique for the probabilistic analysis of the structural performance and has the major advantage of speed. FPI techniques are orders of magnitude more efficient than Monte Carlo simulation methods. This is especially true in the tails regions of the distribution; that is, at very high or low probabilities since the FPI solution time is independent of the probability level. Conversely, in Monte Carlo simulation methods, the computational time increases with very high or low probability levels. Also, FPI allows evaluation of information that describes the relative importance of each random variable. These sensitivity factors can be a valuable aid in optimizing a design.

In IPACS, the probabilistic assessment of composite structures starts with the identification of uncertain primitive variables at constituent and ply levels. These variables are then selectively perturbed several times to create a data base. The data base is used to establish the relationship between the desired structural response (or the desired material property) and the primitive variables. For every given perturbed variable, micromechanics is

applied to determine the corresponding perturbed mechanical properties at the ply and laminate levels. Laminate theory is then used to ascertain the resultant force-moment-strain-curvature relationship. With this relationship at the laminate level, a finite element perturbation analysis is performed to find the structural response that corresponds to the selectively perturbed primitive variables. This process is repeated until enough data are generated and the proper relationship between structural response and primitive variables can be established through a numerical procedure.

Given the probabilistic distributions of primitive variables and a numerically determined relationship between them and the structural response, FPI is applied. For every discrete response value, a corresponding cumulative probability can be computed quickly by FPI. This process is repeated until the CDF can be appropriately represented. The probabilistic material properties at ply and laminate levels are also computed in this same way. The output information from FPI for a given structural response includes its discrete CDF values, the coefficients for the PDF that was used for the uncertainties in the primitive variables, and the variables' sensitivity factors to the structural response. It is important to restate that the process consists of multiple deterministic analyses using verified structural and composite mechanics computer codes. Therefore, the probabilistic simulations are mechanistically accurate and reflect the uncertainties of the primitive variables in the uncertainties of the structural response.

### Step 3: Decision Making and Redesign

IPACS simulates the PDF of a given structural response, such as buckling load, displacement, local stress, local strength, vibration frequencies, and fatigue life. The probability of a design violation for each criterion can be calculated with these PDF functions. When the failure probability is greater than the acceptable value, say  $10^{-3}$ , the composite structural design should be rejected. To redesign a composite structure, one can use the sensitivity factors from the IPACS analysis. Sensitivity factors rank the random variables based on their contribution to this failure probability. Therefore, a redesign will be guided by this information with manufacturing control of the mean and the standard deviation (stdv) of the appropriate random variables.

## DEMONSTRATION CASE AND DISCUSSION

A stiffened composite cylindrical pipe is probabilistically assessed against probabilistic design criteria. The cylindrical pipe is 2 ft in diameter and 20 ft long (fig. 4). The structure is modeled by 588 four-noded shell elements and 600 active nodes (6 degrees of freedom per node). The composite pipe consists of the skin, three horizontal circumferential frames, and four vertical stringers. The laminate configurations for the skin, frames, and stringers are  $[\pm 45/0_2/\pm 45/0_2/\pm 45/0/90]_s$ ,  $[0_{24}]$  and  $[0_{24}]$ , respectively. The pipe is assumed to be supported at one end by a set of translational and torsional spring constants and free at the other end. When the spring constant approaches infinity, a completely fixed boundary condition is simulated. When the spring constants are set to zero, a free boundary condition is simulated. For a given set of spring constants, a partially fixed boundary condition is modeled. The pipe is subjected to axial ( $F_x$ ) and lateral ( $F_y$ ) loads as well as torsional moments ( $M_{xx}$ ) at its free end (fig. 5).

The uncertain variables are identified at the constituent, ply, and structure levels. At the constituent level, 17 material properties for the graphite fiber and 12 for the epoxy matrix of the skin, frames, and stringers are assumed to be uncertain variables. Their probability distribution types and associated parameters are listed in table I. At the ply level, the fabrication variables (fiber and void volume ratio, ply orientation, and ply thickness) are treated as random variables. Their statistics are shown in table II. At the structure level, spring constants that simulate a partially fixed boundary condition are assigned by a probability distribution as are the loading conditions. Their statistics are shown in table III.

In the following paragraphs, the composite pipe is assessed or checked against two design criteria: clearance and delamination; and the results are discussed.

### Clearance Assessment

The clearance criterion is violated when the displacement at the free end in the lateral direction is greater than the allowable value. In this assessment, acceptable failure probability is chosen to be  $10^{-3}$ . From the static analysis, the probabilistic displacement at the free end in the lateral direction was simulated as shown in figure 6. The critical displacement corresponding to  $10^{-3}$  failure probability is 5.2 in. If the allowable displacement is 6 in., then the critical displacement falls in the safe region, and the clearance criterion is satisfied. If the allowable displacement is 5 in., then the critical displacement falls in the failure region. The clearance design criterion is violated and the pipe needs to be redesigned. From the IPACS sensitivity analysis, the fiber modulus, fiber volume ratio, ply thickness of the skin, and random loads in lateral direction have the most significant contribution to the failure probability (fig. 7).

### Delamination Assessment

Delamination occurs when the ply stress is greater than the ply delamination strength. From the IPACS analysis, the relationship between the ply stress  $S_{pl}$  and the independent random variables  $\mathbf{X}$  is numerically determined as shown in equation (1).

$$S_{pl} = a_0 + \sum_{i=1}^N a_i X_i + \sum_{i=1}^N b_i X_i^2 \quad (1)$$

where  $a_0$ ,  $a_i$  and  $b_i$  are constants;  $N$  is number of independent random variables. The ply delamination strength  $S_{DL}$  (ref. 9) is shown in equation (2).

$$S_{DL} = 10 S_1 + 2.5 S_{mT} A \quad (2)$$

$$S_1 = \left[ 1 - \left[ \sqrt{fvr} - fvr \right] \left[ 1 - \frac{G_m}{G_{f12}} \right] \right] S_{mS} A \quad (3)$$

and

$$A = 1 - \sqrt{\frac{4vvr}{\pi [1 - fvr]}} \quad (4)$$

where  $fvr$  and  $vvr$  are fiber volume ratio and void volume ratio;  $G_{f12}$  and  $G_m$  are fiber and matrix shear modulus;  $S_{mT}$  and  $S_{mS}$  are the matrix tensile and shear strength. A limit state function (LSF) is defined as

$$LSF = S_{DL} - S_{pl} \quad (5)$$

If the LSF is less than 0, it indicates that the ply stress is greater than the ply delamination strength. Therefore, delamination will occur. The probability of  $LSF \leq 0$ , computed by FPI, is 0.0008, which is smaller than the acceptable failure probability ( $10^{-3}$ ). From the sensitivity analysis, the eight most influential random variables that contribute to the failure probability are identified in figure 8. The matrix shear strength is the most important (sensitivity factor about 0.8) followed by the ply thickness of the composite skin (sensitivity factor about 0.4). If the acceptable failure probability is reduced to  $10^{-4}$ , redesign is necessary. The redesign can be achieved most efficiently by manufacturing tolerance control or by controlling the mean or scatter of the significant primitive variables. For example, if the coefficient of variation (scatter) of the ply shear strength is reduced from 5 to 4 percent, the failure probability is reduced to 0.0003. However, if the scatter of the ply thickness is also reduced from 5 to 4 percent, the failure probability can only be reduced to 0.0006. This demonstrates that the failure probability can be reduced more effectively by reducing the scatter of the ply shear strength, the most important random variable.

Any other structural response can be similarly evaluated. These two examples demonstrate how the probabilistic design assessment is performed by using IPACS.

### CONCLUDING REMARKS

A formal methodology is described in this report for the probabilistic design assessment of composite structures. This methodology, integrating micro- and macrocomposite theory, structural mechanics (finite element methods), and probability algorithms, performs a probabilistic assessment of composite structural designs considering all identifiable uncertain variables at all composite levels. Composite structural designs can be assessed against specific probabilistic design criteria demonstrating that such designs can be computationally assessed by using the probabilistic computer code IPACS. Information for an efficient design can also be obtained. Specifically, for the demonstration case, the uncertainty range in the end displacement was between 1 and 3 percent of the pipe length and was most sensitive to the uncertainties in the skin-related primitive variables. Conversely, probable delamination failure was most sensitive to the shear strength of the skin.



## APPENDIX—SYMBOLS

|                |  |
|----------------|--|
| $C_f$          | fiber heat capacity, Btu/in. °F  |
| $C_m$          | matrix heat capacity, Btu/in. °F   |
| $D_m$          | matrix diffusivity, in. <sup>3</sup> /sec  |
| $d_f$          | filament equivalent diameter, in.  |
| $E_{f11}$      | fiber modulus in longitudinal direction, Mpsi  |
| $E_{f22}$      | fiber modulus in transverse direction, Mpsi  |
| $E_m$          | matrix elastic modulus, Mpsi   |
| $F_x$          | axial loads, kip   |
| $F_y$          | lateral loads, kip   |
| $fvr$          | fiber volume ratio   |
| $G_{f12}$      | in-plane fiber shear modulus, Mpsi   |
| $G_{f23}$      | out-of-plane fiber shear modulus, Mpsi   |
| $G_m$          | matrix shear modulus, Mpsi   |
| $K_{cTR}$      | translational spring constant, lb/in.  |
| $K_{cTO}$      | torsional spring constant, lb-in./rad  |
| $K_{f11}$      | fiber heat conductivity in longitudinal direction, Btu·in./hr in. <sup>2</sup> °F            |
| $K_{f22}$      | fiber heat conductivity in in-plane transverse direction, Btu·in./hr in. <sup>2</sup> °F     |
| $K_{f33}$      | fiber heat conductivity in out-of-plane transverse direction, Btu·in./hr in. <sup>2</sup> °F |
| $K_m$          | matrix heat conductivity, Btu·in./hr in. <sup>2</sup> °F                                     |
| $M_{xx}$       | torsional moment, kip-ft   |
| $N_f$          | number of fibers per end   |
| $S_{DL}$       | ply delamination strength, ksi   |
| $S_{fC}$       | fiber compressive strength, ksi  |
| $S_{fT}$       | fiber tensile strength, ksi  |
| $S_{mC}$       | matrix compressive strength, ksi   |
| $S_{mS}$       | matrix shear strength, ksi   |
| $S_{mT}$       | matrix tensile strength, ksi   |
| $S_{pl}$       | ply stress   |
| $stdv$         | standard deviation   |
| $t_{psk}$      | ply thickness of skin, in.   |
| $t_{pst}$      | ply thickness of stringer, in.   |
| $vvr$          | void volume ratio  |
| $X$            | vector of independent random variables   |
| $\alpha_{f11}$ | fiber thermal expansion coefficient in longitudinal direction, ppm/ °F                       |
| $\alpha_{f22}$ | fiber thermal expansion coefficient in transverse direction, ppm/ °F                         |
| $\alpha_m$     | matrix thermal expansion coefficient, ppm/ °F  |
| $\beta_m$      | matrix moisture coefficient, in./in.   |
| $\theta_p$     | ply misalignment, deg  |
| $\nu_{f12}$    | in-plane fiber Poisson's ratio   |
| $\nu_{f23}$    | out-of-plane fiber Poisson's ratio   |
| $\nu_m$        | matrix Poisson's ratio   |
| $\rho_f$       | fiber mass density, lb/in. <sup>3</sup>  |
| $\rho_m$       | matrix mass density, lb/in. <sup>3</sup>   |

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TABLE I.—MATERIAL PROPERTIES AT THE CONSTITUENT  
LEVEL FOR SKIN AND STRINGERS

| Property                                   | Assumed<br>distribution<br>type | Mean <sup>a</sup> | Assumed<br>uncertainty<br>scatter |
|--|---------------------------------|-------------------|-----------------------------------|
| $E_{f11}$ , Mpsi                           | Normal                          | 31.0              | 0.05                              |
| $E_{f22}$ , Mpsi                           | ↓                               | 2.0               | ↓                                 |
| $G_{f12}$ , Mpsi                           |                                 | 2.0               |                                   |
| $G_{f23}$ , Mpsi                           |                                 | 1.0               |                                   |
| $\nu_{f12}$                                |                                 | 0.2               |                                   |
| $\nu_{f23}$                                |                                 | .25               |                                   |
| $\alpha_{f11}$ , ppm/ °F                   |                                 | -.55              |                                   |
| $\alpha_{f22}$ , ppm/ °F                   |                                 | 5.6               |                                   |
| $\rho_f$ , lb/in. <sup>3</sup>             |                                 | .063              |                                   |
| $N_f$                                      | Constant                        | 10 000            |                                   |
| $d_f$ , in.                                | Normal                          | .0003             |                                   |
| $C_f$ , Btu/in. °F                         | ↓                               | .17               |                                   |
| $K_{f11}$ , Btu·in./hr in. <sup>2</sup> °F |                                 | 580               |                                   |
| $K_{f22}$ , Btu·in./hr in. <sup>2</sup> °F |                                 | 58                |                                   |
| $K_{f33}$ , Btu·in./hr in. <sup>2</sup> °F |                                 | 58                |                                   |
| $S_{fT}$ , ksi                             | Weibull                         | 400               |                                   |
| $S_{fC}$ , ksi                             | Weibull                         | 400               |                                   |
| $E_m$ , Mpsi                               | Normal                          | .5                |                                   |
| $G_m$ , Mpsi                               | ↓                               | .185              |                                   |
| $\nu_m$                                    |                                 | .35               |                                   |
| $\alpha_m$ , ppm/ °F                       |                                 | 42.8              |                                   |
| $\rho_m$ , lb/in. <sup>3</sup>             |                                 | .0443             |                                   |
| $C_m$ , Btu/in. °F                         |                                 | .25               |                                   |
| $K_m$ , Btu·in./hr in. <sup>2</sup> °F     | ↓                               | 1.25              |                                   |
| $S_{mT}$ , ksi                             | Weibull                         | 15                |                                   |
| $S_{mC}$ , ksi                             | Weibull                         | 35                |                                   |
| $S_{mS}$ , ksi                             | Weibull                         | 13                |                                   |
| $\beta_m$ , (in./in.)/1% moist             | Normal                          | .004              |                                   |
| $D_m$ , in. <sup>3</sup> /sec              | Normal                          | .002              | ↓                                 |

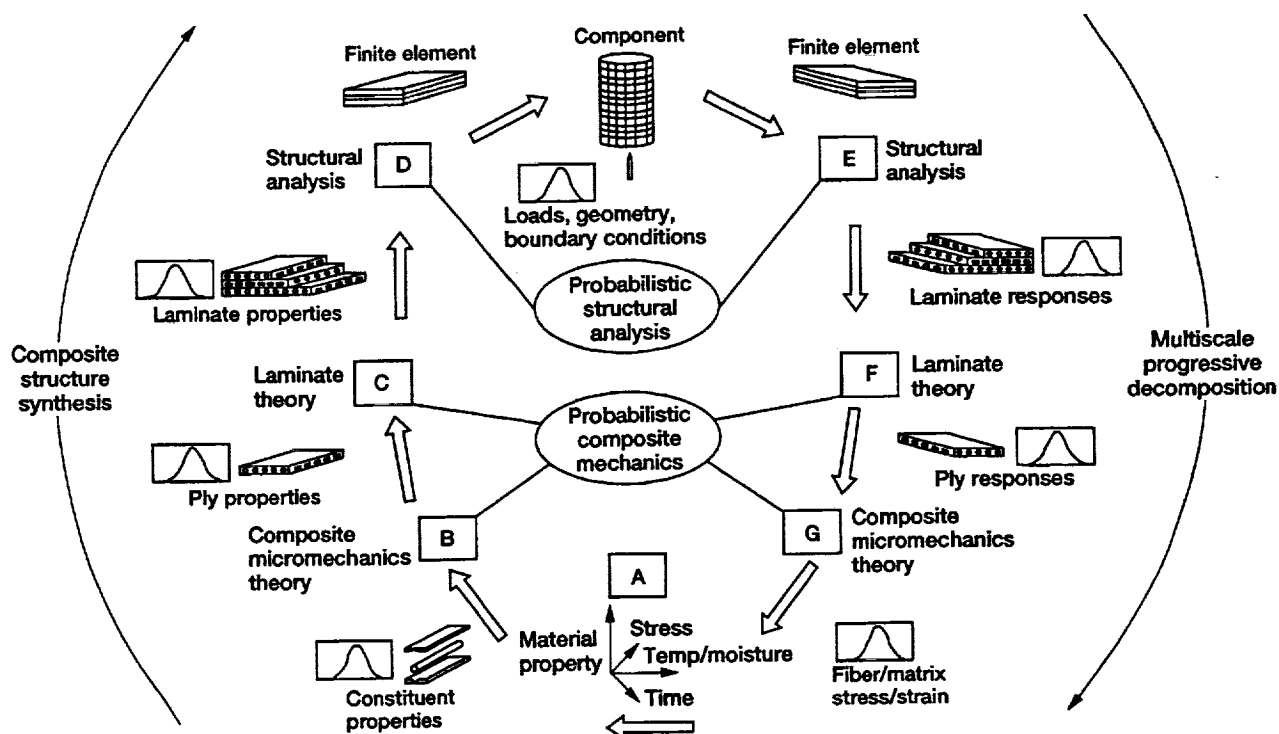
<sup>a</sup>Typical values for graphite-fiber/epoxy-matrix composites at 0.6 fiber  
volume ratio.

TABLE II.—FABRICATION VARIABLES  
AT PLY LEVEL

| Variable         | Assumed<br>distribution<br>type | Mean | Assumed<br>uncertainty<br>scatter |
|------------------|---------------------------------|------|-----------------------------------|
| fvr              | Normal                          | 0.60 | 0.05                              |
| vvr              | ↓                               | .02  | .05                               |
| $\theta_p$ , deg |                                 | .00  | .9 (stvd)                         |
| $t_{psk}$ , in.  |                                 | .005 | .05                               |
| $t_{pst}$ , in.  | ↓                               | .02  | .05                               |

### TABLE III—UNCERTAINTIES IN STRUCTURAL LEVEL

| Uncertainty            | Assumed distribution type | Mean             | Assumed uncertainty scatter |
|------------------------|---------------------------|------------------|-----------------------------|
| $K_{CTR}$ , lb/in.     | Normal<br>↓               | $30 \times 10^6$ | 0.20                        |
| $K_{CTO}$ , lb-in./rad |                           | $12 \times 10^2$ | .20                         |
| $F_x$ , kip            |                           | 288              | .05                         |
| $F_y$ , kip            |                           | 5.76             | .05                         |
| $M_{rx}$ , kip-ft      |                           | 576              | .05                         |



**Figure 1.—Concept of probabilistic assessment of composite structures.**

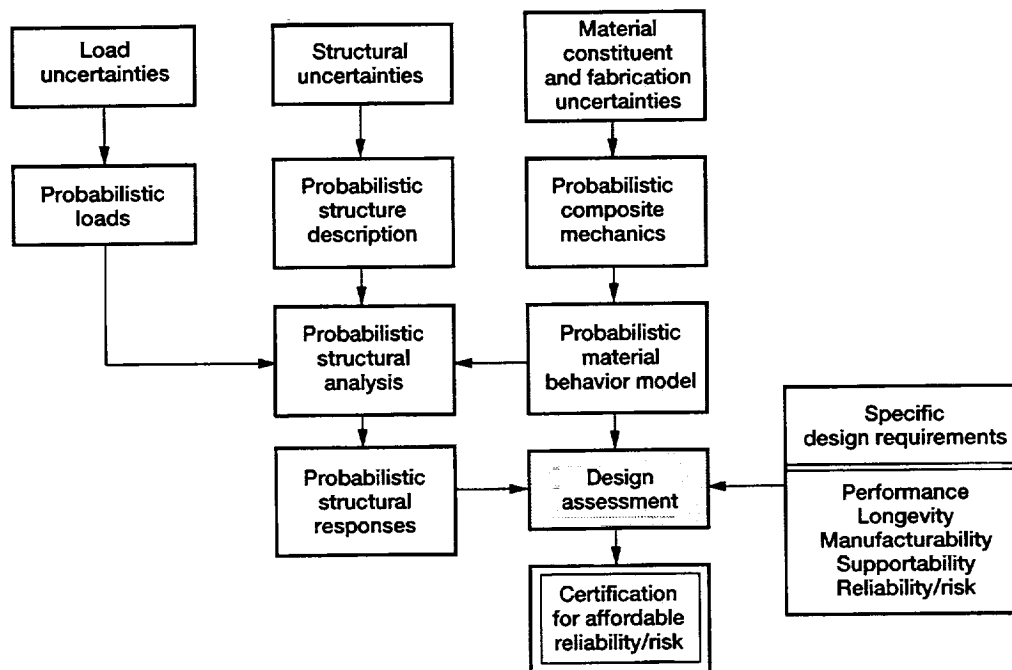


Figure 2.—Probabilistic design assessment of composite structures.

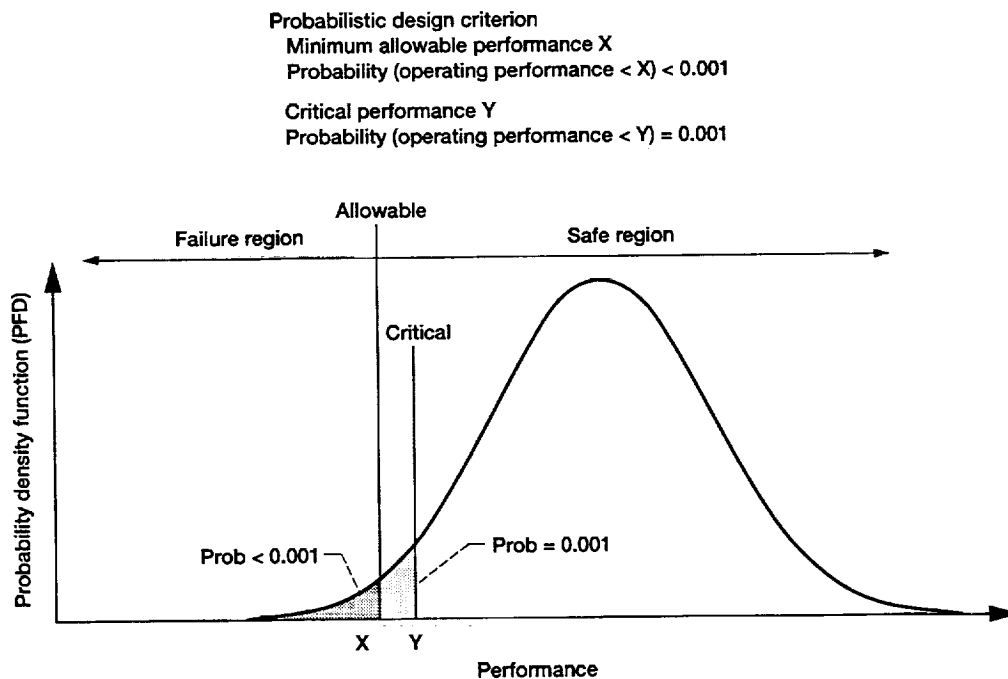
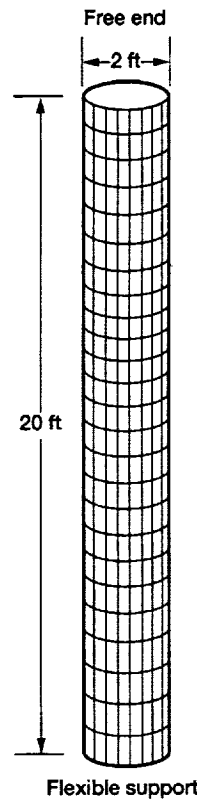


Figure 3.—Safe and failure regions in a probability space.



Skin  
cylindrical shell  
[ $\pm 45/0_2/\pm 45/0_2/\pm 45/0/90$ ]<sub>s</sub>

Circumferential frame (horizontal)  
three T-shape frames (10 ft apart)  
[0<sub>2</sub>4]

Stringer (vertical)  
four T-shape stringers (90 deg apart)  
[0<sub>2</sub>4]

Thickness  
ply thickness 0.005 in.

Figure 4.—Geometry and composite configuration of stiffened composite pipe.

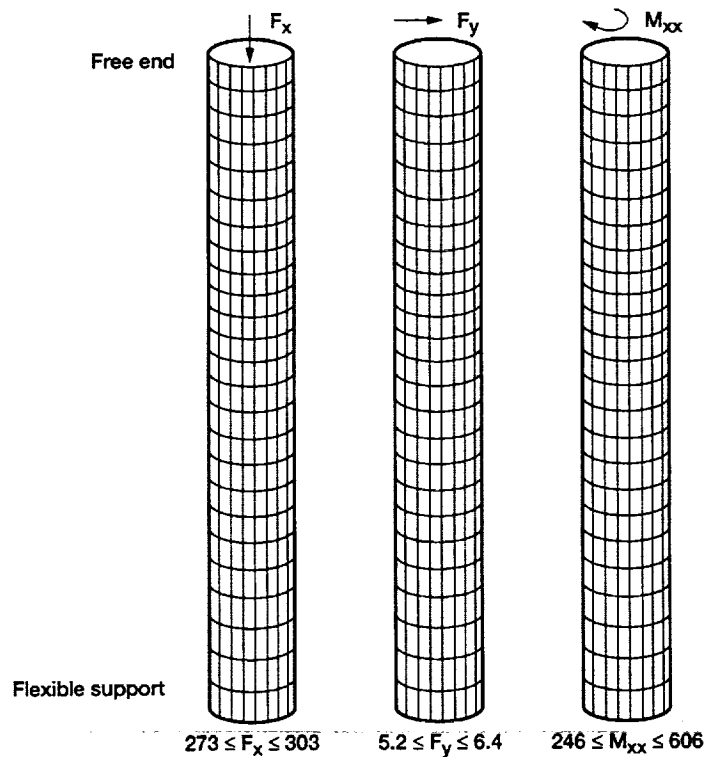


Figure 5.—Probabilistic loading conditions (kip) of stiffened composite pipe.

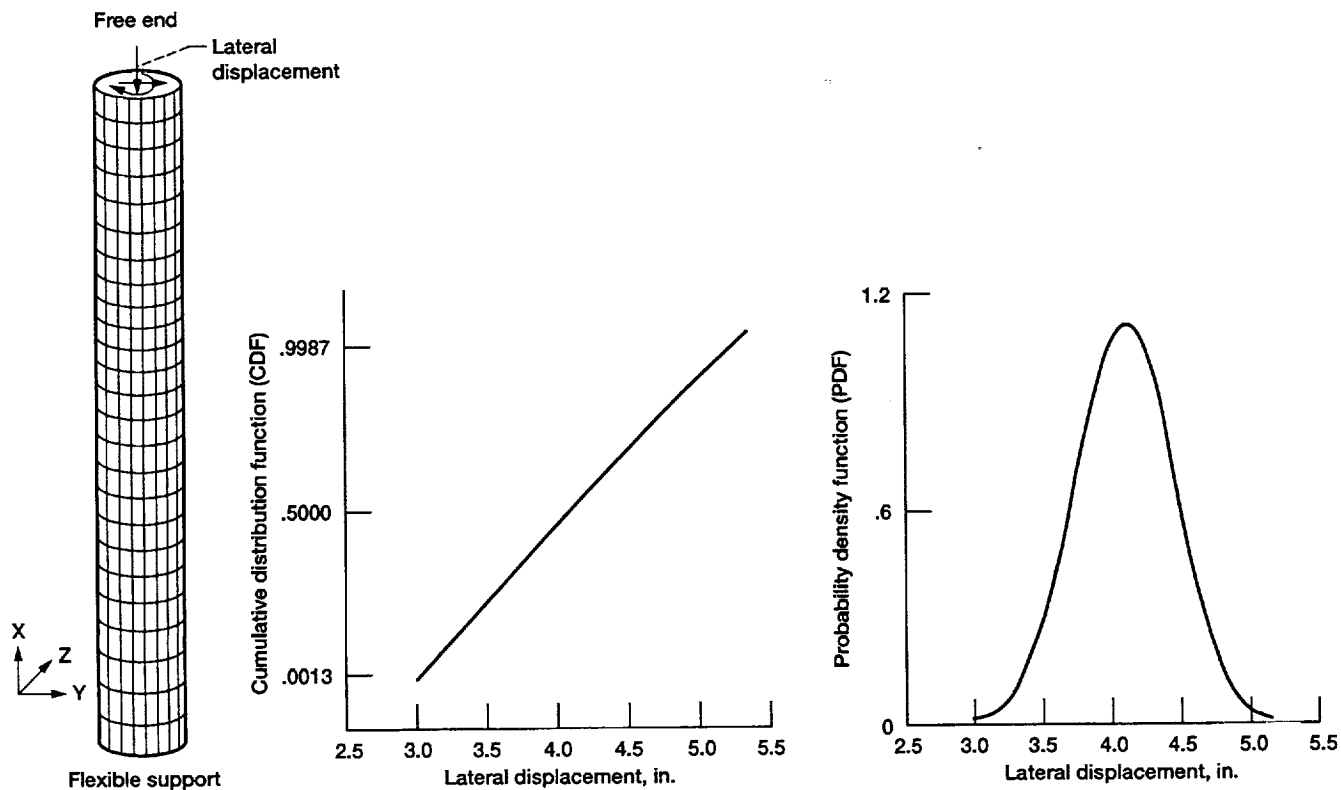


Figure 6.—Cumulative distribution function (CDF) and probability density function (PDF) of lateral displacement at free end.

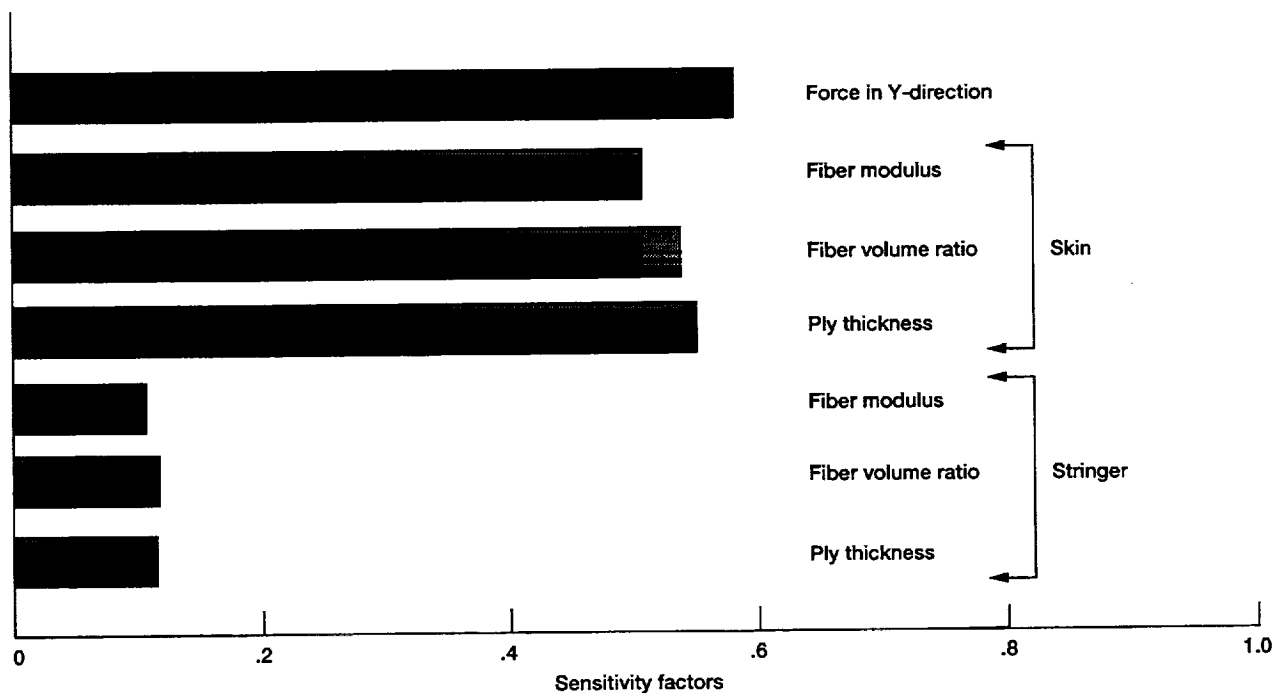


Figure 7.—Sensitivity factors of lateral displacement at free end at 0.001 failure probability.

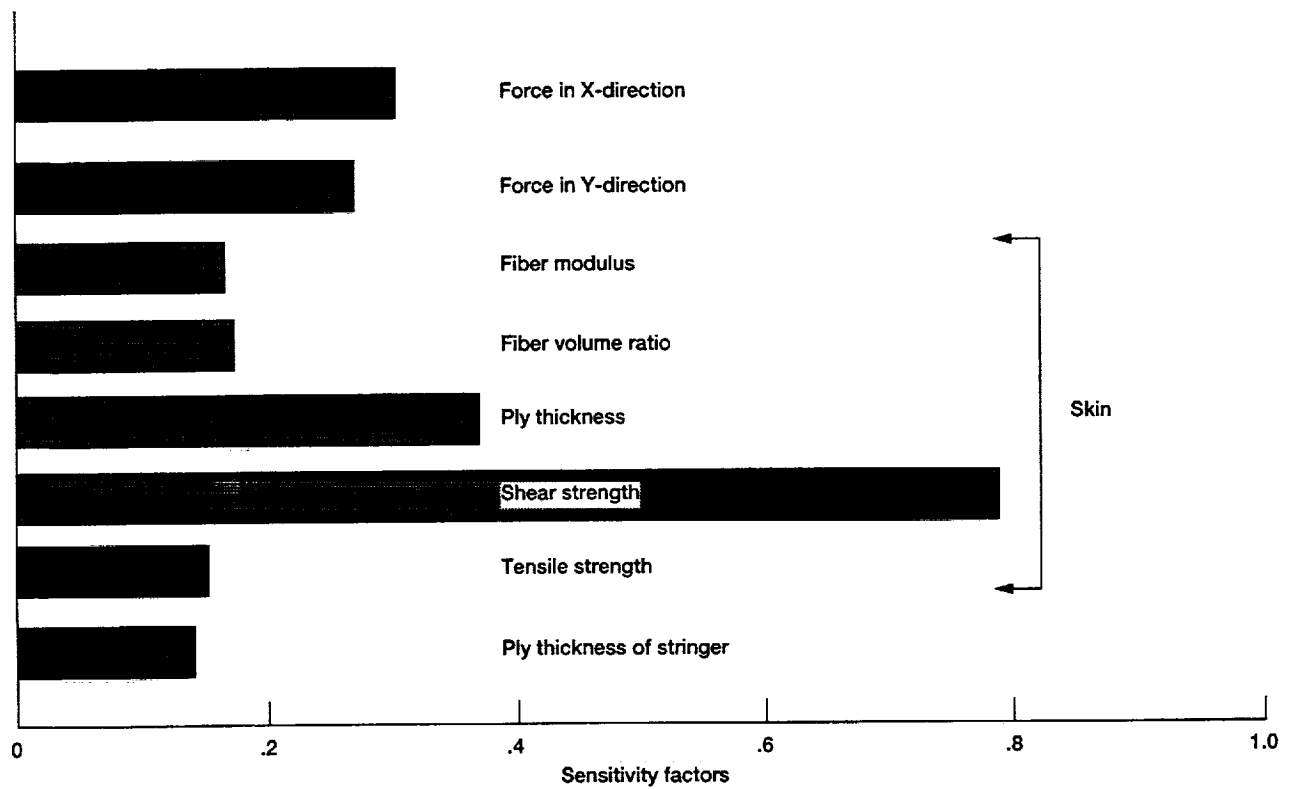


Figure 8.—Sensitivity factors for 0.0008 delamination probability.





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